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Paris, 1796, pp. 194–195. Also "Dilucidationes super formulis, quibus sinus et cosinus angulorum multiplorum exprimi solent, ubi simul ingentes difficultates diluuntur," *Nova acta acad. sc. Petrop.*, volume 9 (1791), 1795, pp. 54–55; "Conventui exhib. die 6 Mart. 1777.") In a posthumous paper, "Enodatio insignis cujusdam paradoxi circa multiplicationem angulorum observati," (*Opera postuma*, vol. 1, 1862, pp. 299–314) Euler discussed cos $n\phi$ as a power series in cos ϕ.

The developments of sin nx and cos nx into series in sin x and cos x, when n is not restricted to integral values, seem to have been first thoroughly discussed by Poinsot and forms the subject of his *Recherches sur l'Analyse des Sections Angulaires*, Paris, 1825, 80 quarto pages. ARC.

SOLUTIONS OF PROBLEMS.

417 (Algebra) [1914, 156]. Proposed by A. J. RICHARDSON, Marquette, Mich.

Required to reduce the quartic

$$x^4 + px^2 + qx + r = 0$$

to the form

$$(x^2 + k)^2 = [(2k - p)x + (2k - q)]^2,$$

wherein k is the solution of a certain cubic.[1] Hence, express the solution of the given quartic in terms of p, q, r, and k.

SOLUTION BY H. S. UHLER, Yale University.

We shall show that the hypothesis that the given quartic can be reduced to the required form destroys the mutual independence of the coefficients p, q, r, and that k is not "the solution of a certain cubic."

Transposing the right-hand member of

$$(x^2 + k)^2 = [(2k - p)x + (2k - q)]^2,$$

expanding, and collecting terms, we find

$$x^4 + [2k - (2k - p)^2]x^2 - 2(2k - p)(2k - q)x + [k^2 - (2k - q)^2] = 0.$$

Since, by hypothesis, the last equation is a modified form of the given quartic the coefficients of like powers of x in the two equations must be equal, that is, the three following conditions must hold

$$2k - (2k - p)^2 = p, \tag{1}$$

$$-2(2k - p)(2k - q) = q, \tag{2}$$

$$k^2 - (2k - q)^2 = r. \tag{3}$$

Condition (1) gives

$$2k - p = 0, \tag{4}$$

or

$$2k - p = 1. \tag{5}$$

In either case k is a linear function of p alone and hence k is not dependent upon the solution of any cubic.

Combining relations (2) and (4) we see that q must have the special value zero. Then the given quartic reduces to a quadratic in x^2 and the solution is immediate.

[1] There must have been some mistake in writing this problem. The following may have been intended:

$$(x^2 + k)^2 = [x\sqrt{2k - p} - \sqrt{k^2 - r}]^2.$$

See Todhunter, *An Elementary Treatise on the Theory of Equations*, London, 1880, p. 117.—EDITORS.

Combining relations (2) and (5) we obtain

$$k = q/4. \tag{6}$$

Equating the values of k given by relations (5) and (6) we find that p and q are connected by the linear equation

$$2p - q + 2 = 0. \tag{7}$$

Substituting k from relation (6) in relation (3) and then employing equation (7) we find

$$r = -3q^2/16 = -3(p+1)^2/4. \tag{8}$$

Incidentally, the last result shows that r must be negative in order that p and q may be real.

Relations (7) and (8) enable the given quartic to be expressed in terms of (say) p alone, namely

$$x^4 + px^2 + 2(p+1)x - 3(p+1)^2/4 = 0,$$

or

$$[x^2 + (p+1)/2]^2 = [x - (p+1)]^2.$$

Consequently, the roots of the quartic are

$$x = (1 \pm \sqrt{-6p-5})/2,$$

$$x = (-1 \pm \sqrt{2p+3})/2.$$

For $-3/2 \leqq p \leqq -5/6$ all four roots will be real. For values of p outside of this range two roots will be real and two complex.

The reduction of the general quartic to the difference of two squares is given in Burnside and Panton's *Theory of Equations*, vol. I, p. 129, Art. 63 (4th or 7th editions.)

2783 [1919, 312]. Proposed by C. C. BRAMBLE, U. S. Naval Academy.

Two players, A and B, take turns throwing a single dice, A leading. The one first making a score of three aces is to be the winner. Find the probability that A will win.

SOLUTION BY H. P. MANNING, Brown University.

The probability of throwing three aces in the first three throws is $1/6^3$; in general, the probability of throwing exactly two aces in the first n throws, and an ace in the $(n+1)$th throw, is $\frac{n(n-1)}{2} \cdot \frac{5^{n-2}}{6^{n+1}}$. That is, the probabilities of the different ways of throwing the first three aces are for A or B

$$\frac{1}{6^3}, \quad 3\frac{5}{6^4}, \quad 6\frac{5^2}{6^5}, \quad \cdots.$$

Call these numbers

$$a \quad b \quad c \quad \cdots.$$

Their sum should equal 1.

The probability that A wins is

$$a + (1-a)b + (1-a-b)c + \cdots,$$

the factor in the parenthesis in any term indicating the probability that B has not yet thrown three aces. This expression may be written

$$a + b + c + \cdots$$
$$- (ab + ac + bc + \cdots).$$

Similarly the probability that B wins is

$$(1-a)a + (1-a-b)b + \cdots,$$

which may be written

$$a + b + c + \cdots$$
$$- (a^2 + ab + b^2 + ac + bc + c^2 + \cdots).$$